Using quantile regression to explore the distribution of ‘Contextual Value Added’ across London

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Outline

- **Looking at Pupil/School Performance using the NPD**
  - The current approach
- **Outline to using quantile or M-quantile approaches**
- **M-quantiles for exploring pupil and school performance**
- **Measuring and mapping performance across local authorities in London**
  - Outcome data for 2006

The National Pupil Database

- **A major Admin database held by DCSF that is utilised by researchers studying pupil and school performance…**
  - Longitudinal record of a pupils’ attendance at State schools in England (updated each term) with performance data linked in at KS1, KS2, (KS3), KS4, and now KS5 with further extensions into further/higher education

- **Limited covariates on individual pupils and their family background**
  - Language, ethnicity, fsm, income deprivation of area, in care,…
  - Possible use of linkage with the LSYPE, which does have the more detailed family background (parental education)…

Measuring School Performance

- **Raw exam scores can be misleading for measuring school performance**
  - Do not reflect different intakes of schools

Value Added (VA) Models:

- **Provide a better measure of performance by accounting for pupil prior attainment**

Contextualised Value Added (CVA) Models:

- **Extension of VA models that also account for pupil characteristics (gender, age, deprivation) and the context within the school**

Current CVA Model

Concept:

- **Include school-specific random effects to account for the between school variation beyond that explained by the variation in model covariates.**
  - Captures the fact that pupil performance within a school is correlated, even after controlling for characteristics

Notation: (s = School, i = Pupil)

- **Variable of interest:** y_{is}
- **Covariate information:** x_{is}
- **School level random effect:** u_s
- **Pupil level random effect:** e_{is}

Current CVA Model (Random Effects Model)

Dependent variable: Capped total Key Stage 4 score (best 8 GCSEs)

Covariates (pupil level):

- **Pupil prior attainment, fsm, income deprivation, special education needs, age, pupil mobility, gender, in care, ethnicity, English as an additional language, interaction terms**

Covariates (school level):

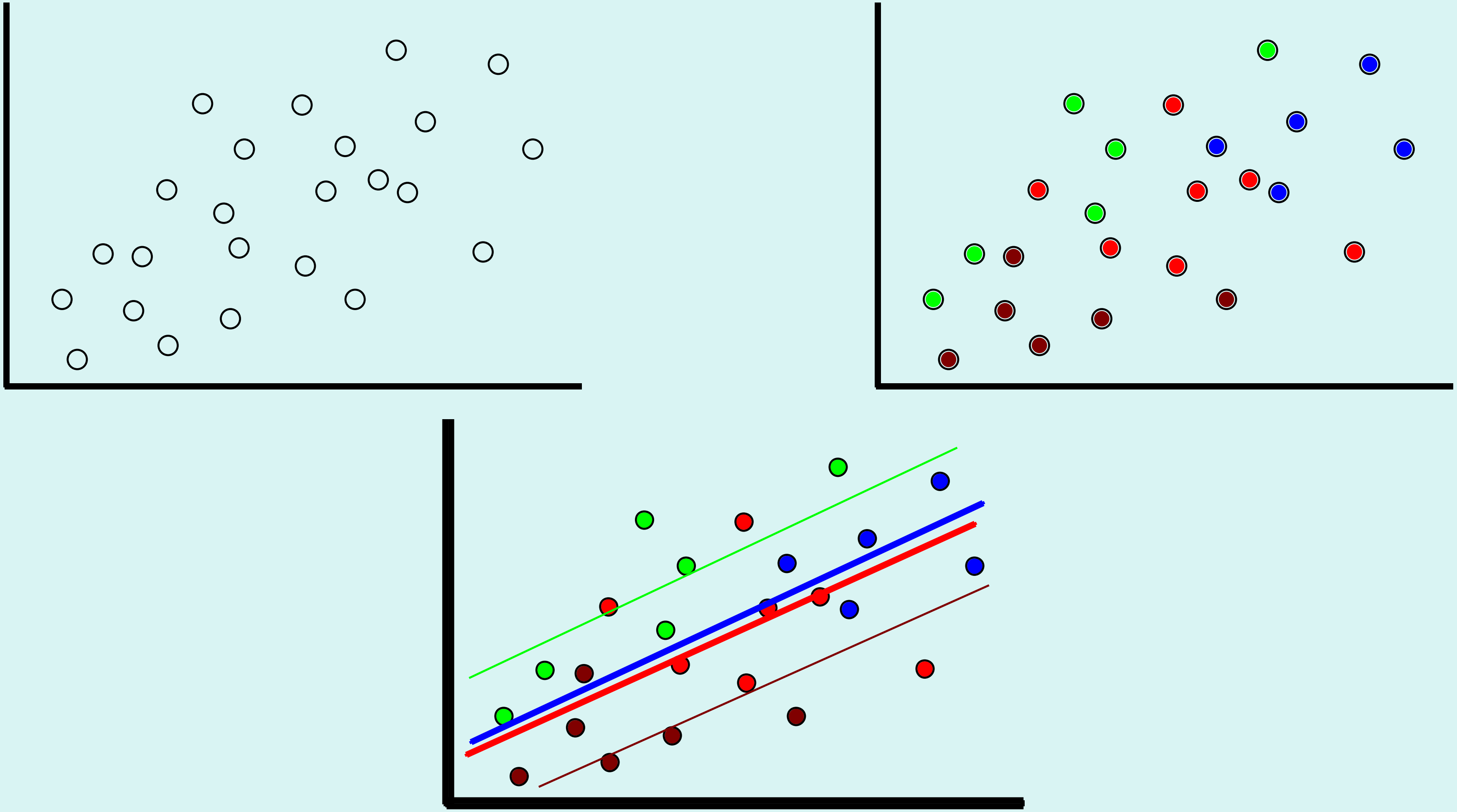
- **School mean prior attainment, school mean spread**

$$y_{is} = \alpha + \beta x_{is} + u_s + e_{is}$$

School random effect u_s:

- **Measures an unknown underlying level of performance for each school**
  - Normally distributed with constant variance…

Random Intercepts



Problems…

- **Why should these (random) school effects be normally distributed with a constant variance?**
  - Problems with capping in high performing schools…

- **Possible lack of outlier robustness**
  - Outlier schools and outlier pupils…

- **The current CVA model assumes a random intercepts specification, what if random slopes provide a better fit?**
  - Losing information by simply summarising the school impact as a single value…
  - *Is the school impact really **the same** for all pupils in the school?*

The Quantile Approach

- **The conventional definition of a regression model as a model for the mean of Y|X can be extended considerably**
  - We view regression analysis as aimed at modelling the entire **conditional distribution** f(Y|X)

- **Regression quantiles, and the easier to compute regression M-quantiles, offer a deeper understanding of the structure of conditional distributions**
  - In this presentation we don't distinguish between regression quantiles and regression M-quantiles as both will serve the same purpose

The Quantile Approach

A Linear Model for Regression Quantiles

$$y_i = \alpha_q + \beta_q x_i + e_i$$

Estimation of Regression Quantiles

Computation: **Simplex Algorithm (quantile regression)**

Weighted Least Squares (M-quantile regression)

Implemented in: **R quantreg library**

Stata qreg

- **For M-quantiles (Chambers and Tzavidis, 2006) we use the rlm function in R modified to qrlm**

Interpreting Regression Quantiles

- **For each value of q, the corresponding model shows how the qth percentile (quantile) of f(Y|X) varies with X**

- **q = 0.5 line shows how the "middle" (median) of f(Y|X) changes with X**

- **q = 0.1 line separates the "top" 90% of f(Y|X) from the "bottom" 10%**
  - it represents the behaviour of units that are "better" than the "worst" 10% and "worse" than the "best" 90%
  - the 'fitted' regression quantiles do not need to be parallel but they should not cross… (*if they do implies poor model specification*)

M-quantile Coefficients

- **Individual level pupil data (y_i, x_i) on Y and X**
- **Linear regression M-quantiles $m_q(x_i) = \alpha_q + \beta_q x_i$**
- **For fixed x, $m_q(x_i)$ is continuous in q**
  - each sample value (y_i, x_i) will lie on **one and only one** regression M-quantile line
- **We refer to the q-value q_i of this regression M-quantile as the M-quantile coefficient or q value of the corresponding pupil**
  - The M-quantile coefficients lie between 0 and 1 and characterize where the pupils lie in the conditional distribution f(Y|X)

Properties of the q_i's

- **The q_i's represent dimensionless measures of the residual heterogeneity in Y after accounting for heterogeneity in X**

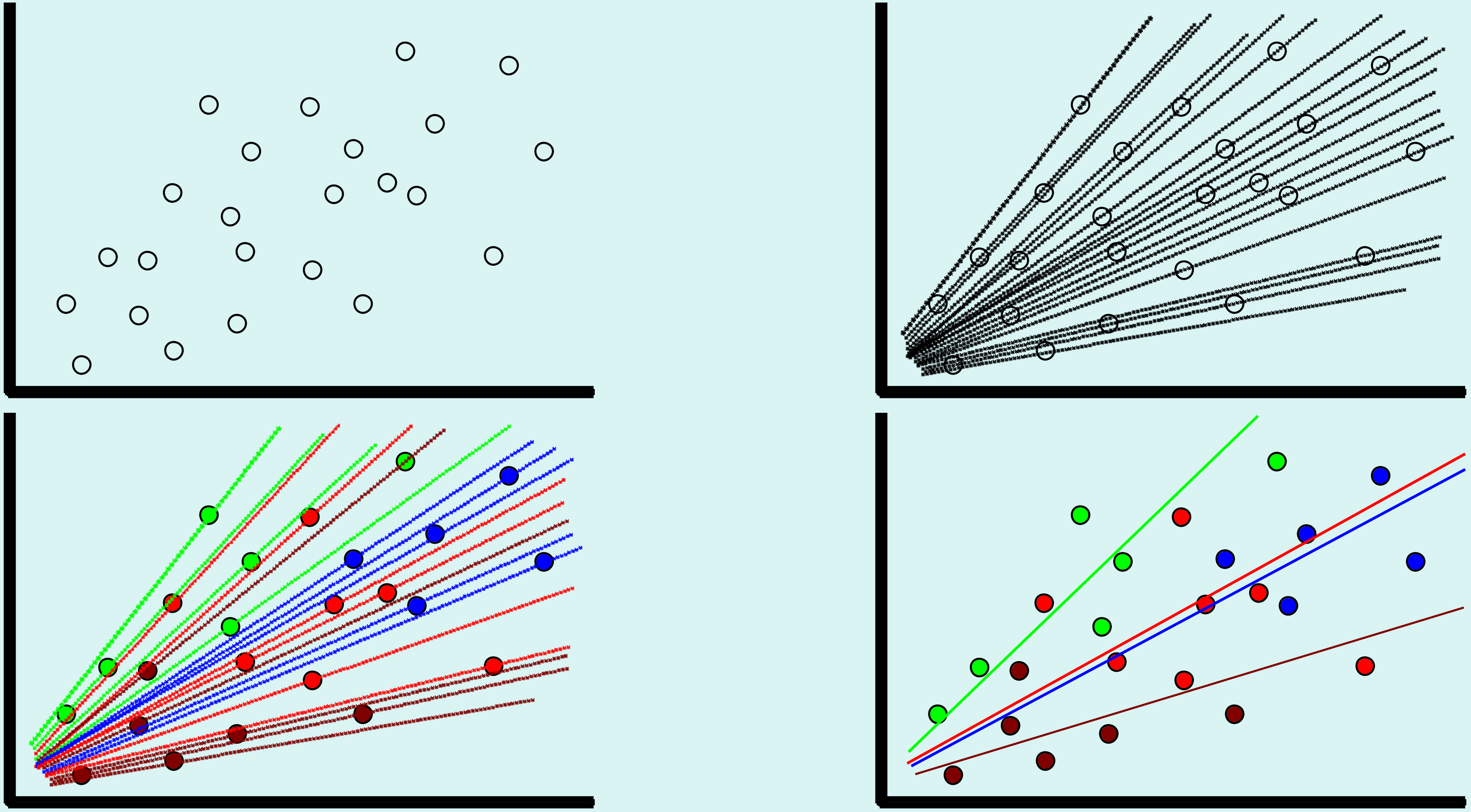
- **The q_i's satisfy 4 conditions that a good measure of performance should satisfy (Kokic et al., 1997)**
  - They lie between 0 and 1
  - The poorest performing pupils given their x's (prior attainment,…) have a performance measure close to zero
  - The best performing pupils have a performance measure close to one
  - The distribution of the performance measure should not depend on the level of inputs x (i.e. the pupil's prior attainment)

Alternative for School Performance…

- **Use M-quantile coefficients to characterise group differences (Chambers and Tzavidis, 2006; Aragon et al., 2006)**
  - **Step 1:** Define a grid of q-values, e.g. g =(0.001,…,0.999) that adequately "covers" the conditional distribution of Y and X
  - **Step 2:** Fit an M-quantile model for each q-value in g and estimate the ***unique*** M-quantile coefficient q_i for each pupil in the sample
  - **Step 3:** The q_i's describe pupil differences after controlling for X.
    - Higher q_i's imply better performance
  - **Step 4:** Using the q_i's of pupils in the same school, estimate a school M-quantile coefficient, Q_s, using the mean or the median

- **Measure of school performance given by Q_s: Higher → Better**

How does it work?



Advantages of using Q_s for School Performance…

- **No normality or constant variance assumptions on the random effects**
  - Should cope better with the capping of the performance measure…

- **No modelling assumptions analogous to random intercepts or random slopes**
  - the data guide the modelling process – OK here as we have a lot of data even if we just consider pupils in London

- **Outlier robustness automatically achieved by using M-quantiles**

MSE Estimation (aggregated effects)

We implement a non-parametric bootstrap (Tzavidis et al. 2010)

- Starting from the original sample s, fit the M-quantile model and compute

$$e_{ij} = y_{ij} - \mathbf{x}_{ij}^T \hat{\boldsymbol{\beta}}_{\psi}(Q_s)$$

- B bootstrap finite populations U^* are generated by sampling e_{ij}^*

$$y_{ij}^* = \mathbf{x}_{ij}^T \hat{\boldsymbol{\beta}}_{\psi}(Q_s) + e_{ij}^*$$

- From each bootstrap population, select L samples using simple random sampling without replacement within the schools

- For each bootstrap sample L implement the procedure for estimating school effects

$$MSE(\hat{p}_j) = B^{-1}L^{-1}\sum_{b=1}^{B}\sum_{l=1}^{L}\left\{\hat{p}_j^{*bl} - \mathrm{av}_L(\hat{p}_j^{*bl})\right\}^2 + \left\{B^{-1}L^{-1}\sum_{b=1}^{B}\sum_{l=1}^{L}\left(\hat{p}_j^{*bl} - p_j^{*b}\right)\right\}^2$$

MSE Estimation

Assessed performance using a design-based simulation

- Fixed population data: NPD/PLASC for London schools with more than 170 pupils i.e. 30,208 pupils nested within 146 schools

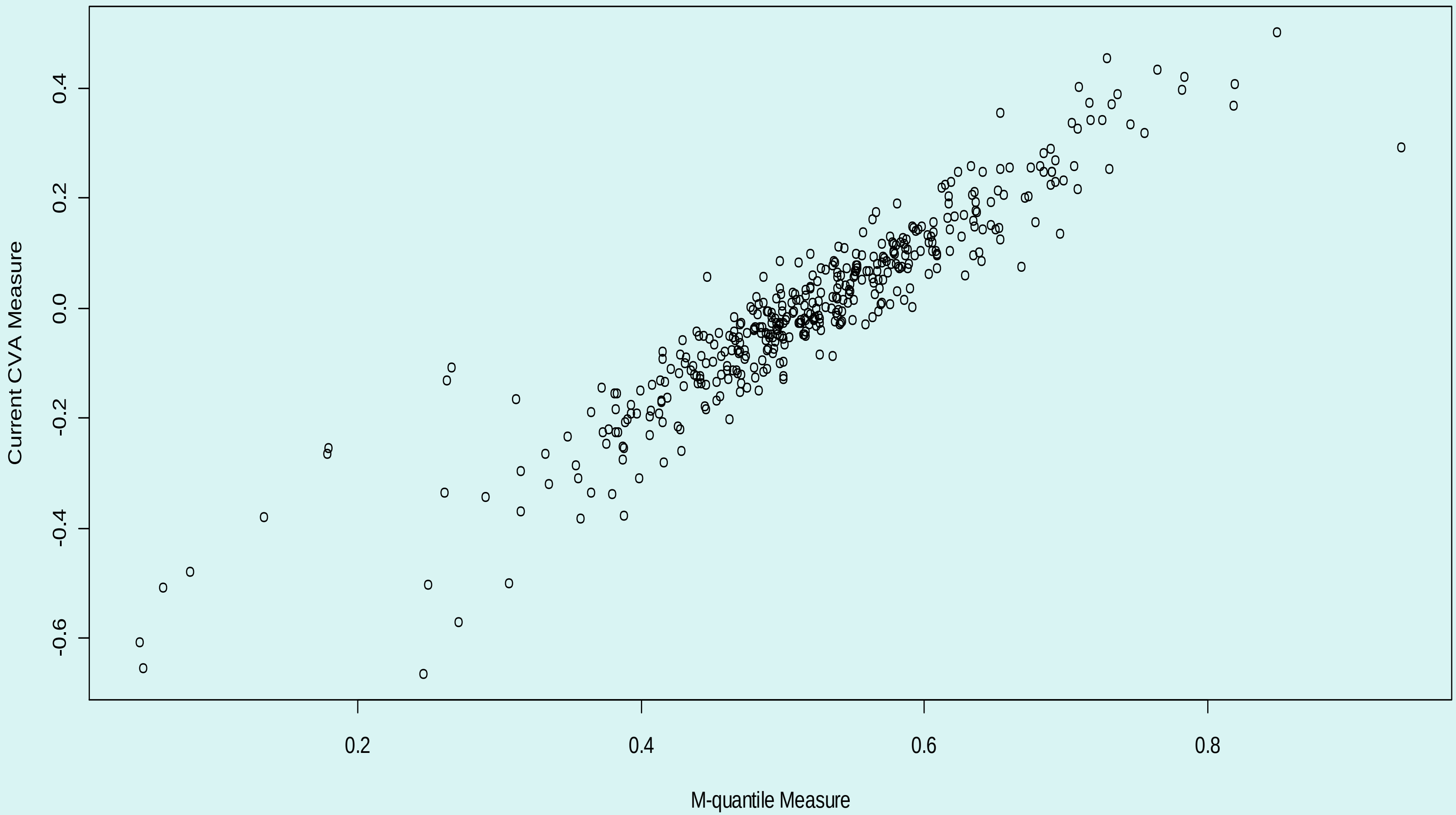
- A total of 500 independent random samples are then taken from the population by randomly selecting 7% of the pupils within schools

- MSE estimation: For each Monte-Carlo sample we implement the bootstrap scheme with B=1 and L=250

Results demonstrate:

- negligible bias in the point estimator of the school effects
- good performance of the MSE estimator (Bias and Coverage)

Comparing School CVA



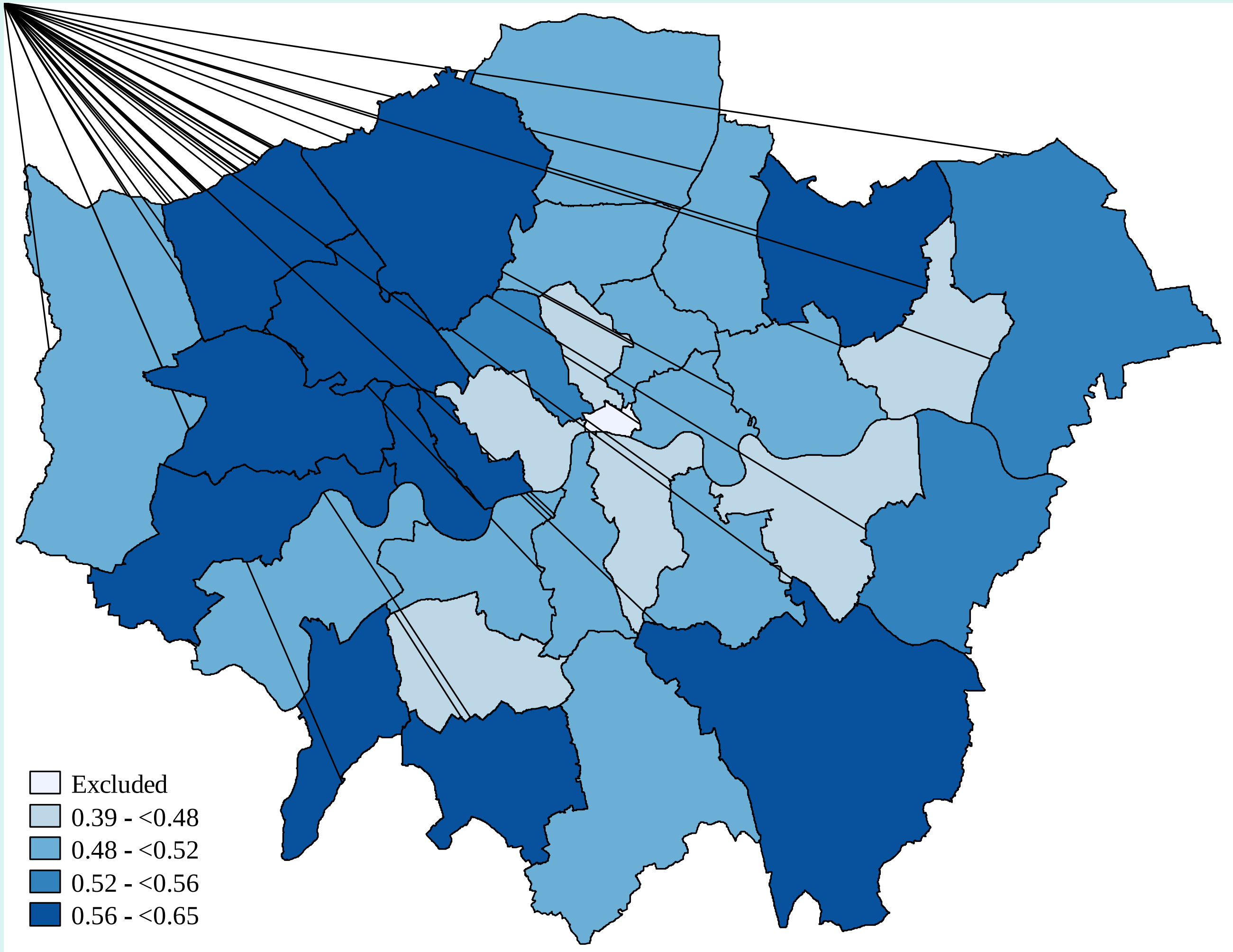
Performance across LAs

- **By estimating a pupil level effect we do not pre-impose a structure on the data**
  - To get a school performance value we are averaging across pupils in the school...
  - The school has an impact when the average differs from 0.5

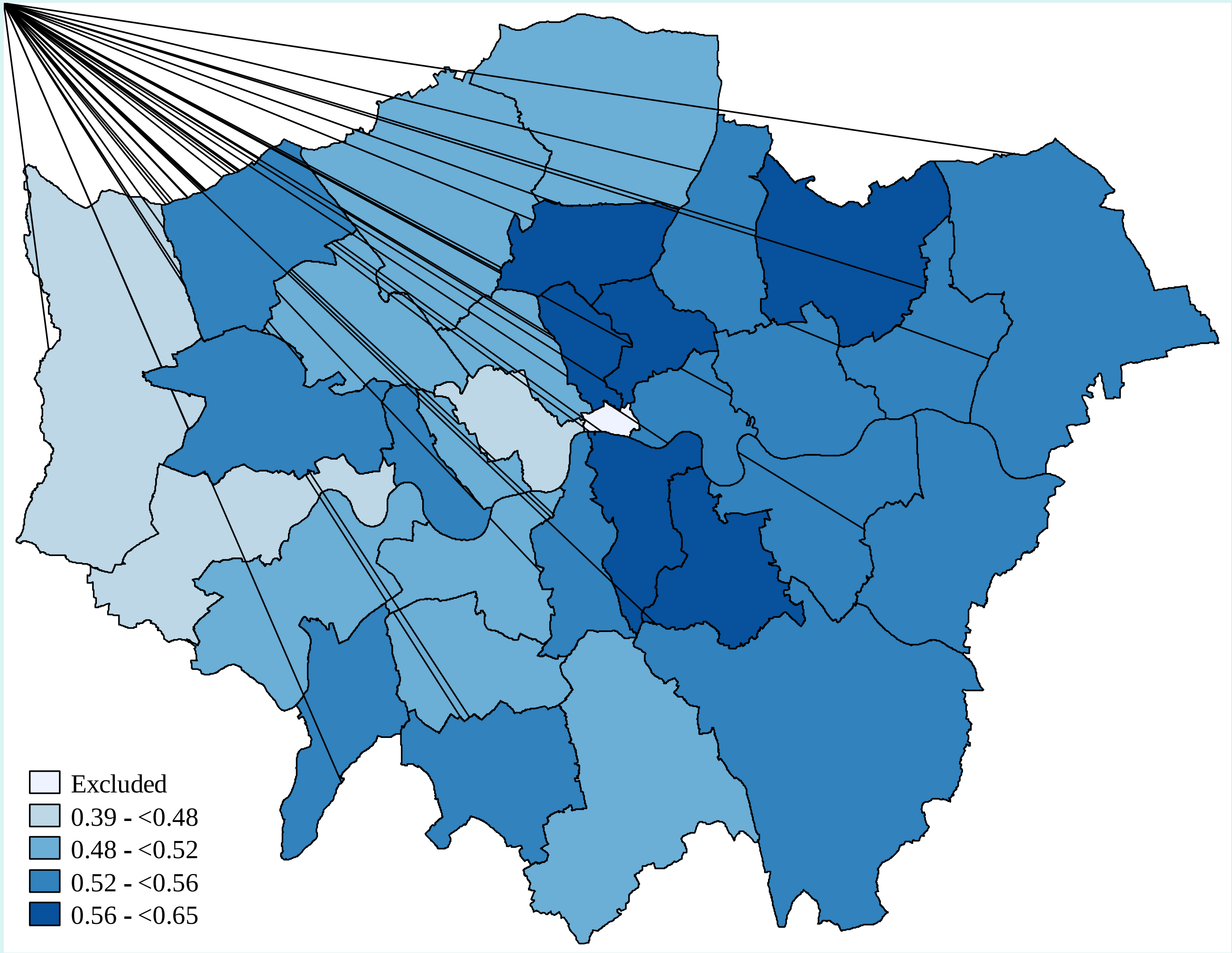
BUT

- **We can estimate for other structures by aggregating our pupils by the desired structure**
  - the impact of the structure being represented by an average efficiency for the pupils different to 0.5

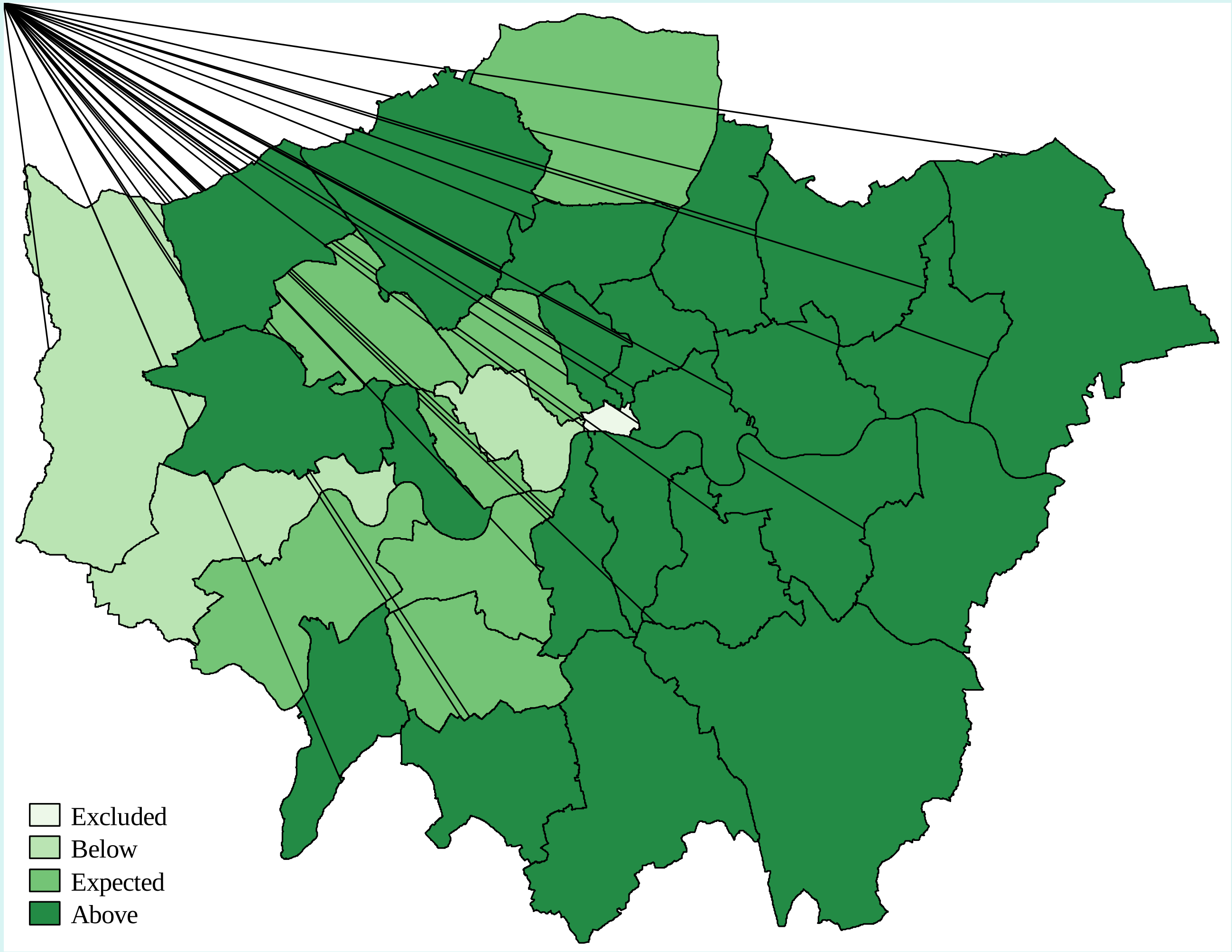
Performance across LAs (marginal measure)



Performance across LAs (conditional measure)



Performance across LAs (difference from 0.5)



Discussion...

- **Utilising quantile models leads to a 'robust' measure of the relative performance of the individual pupils**
  - Measures how efficient a pupil is relative to others with the same inputs (prior attainment) and context

- **We can then impose structure on the data to see if that has any influence on the performance of a group of students**
  - Similar to the multilevel structure in the current CVA model but we are not constrained to a specific structure
  - School effects
  - Average pupil performance at the LA level

Some References
- M-quantiles

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